

Is it *really* that simple?

Honors 320: Great Questions Tutorial

“The bottom line is...” my father began on the phone. I smirked to myself, some 7000 miles away, waiting to hear the nugget of insight that my dating dilemma boiled down to. “The bottom line is she is more interested in pursuing a career right now than following the counsel of her religious leaders and looking for a companion.” I certainly wished that was true; it provided an easy scapegoat for my hurt feelings and frustration at being rejected. But honestly evaluating the situation, I couldn’t assign this single cause as confidently as my father did. The girl I was interested in dating was in a critical recruiting phase of her undergraduate career and probably was just busy. Or maybe she was simply using ‘busyness’ as an excuse to divert my attention. Perhaps there was some speckle of truth in my father’s conclusion, and perhaps it was the “bottom line.” Nevertheless, I was not convinced it was just that simple.

I.

Sometime around the turn of the 19th century, while Carl Gauss was revolutionizing modern mathematics, he discovered an interesting property of complex numbers: a complex product of sums could be calculated with just three, rather than four, terms.¹ That is:

$$(a + bi)(c + di) = ac - bd + (bc + ad)i$$

could be solved with:

$$(a + bi)(c + di) = ac - bd + [(a + b)(c + d) - ac - bd]i.$$

¹ I say ‘sometime’ because I can’t seem to find a definitive date associated with this discovery: see the University of Tennessee timeline from Alex Freire as a general guide.

² As a refresher, ‘*i*’ refers to the imaginary number $\sqrt{-1}$. While a bizarre concept to wrap your head around, it is an essential theoretical component of modern math, engineering, and physics applications.

In other words, Gauss reduced four multiplications in the equation to just three. Admittedly, that does not seem particularly remarkable, and Gauss' new solution visually appears to be more complex than the original equation. However, this obscure insight proved to be significant some 150-odd years later, when Russian mathematician Anatoly Karatsuba used it to design a groundbreaking algorithm for performing computer multiplication more quickly than was previously thought possible.³ Karatsuba took advantage of Gauss's insight by employing an algorithmic approach called *divide-and-conquer*. As the name suggests, this involves taking a problem and breaking it down into smaller, simpler sub-problems of the same type, solving those, and then combining the answers. For Karatsuba's multiplication algorithm, this meant taking a number and splitting it into only three simpler multiplication problems instead of four, and then recombining the results to generate a final answer. Gauss' algebraic expansion (that further complicated an already complex equation) resulted in a significant theoretical breakthrough in computer science.

Ironically, our typical user experience with computers hides all of this complexity under a façade of aesthetic simplicity. When I type in 723×83 into my onscreen calculator, the answer is instantaneous: 60,009. But there is much more going on. Karatsuba's story demonstrates that understanding the "invisible," under-the-hood complexity can reshape our ability to advance technology, which allows you and I, the ignorant consumers, to blissfully sit back and appreciate it with the push of a button.

³ In computer science BigO terms, the traditional multiplication algorithm's time complexity of $\Theta(n^2)$ was improved to $\sim\Theta(n^{1.58})$. As a point of reference, for a computer adding is 'cheap' (linear time $\Theta(n)$), while multiplication is slower (quadratic time, $\Theta(n^2)$), so eliminating a multiplication step can make a big difference.

II.

The 2016 United States presidential race was one that baffled the political world. Even Nate Silver's FiveThirtyEight website, which has a reputation for being statistically rigorous and accurate, reported Hillary Clinton with a 42% higher chance of winning over Donald Trump as late as November 8th.⁴ Various attempts to explain the bizarre outcome have cited the extensive media attention Trump received, a "populist revolt" akin to the Brexit vote in the UK earlier that year, and Trump's platform that appeared to divide the nation along demographic lines. Looking towards his first year as President, one news outlet made the following observation: "...Trump will not be a normal president. He won the White House by waging one of the most divisive and polarizing campaigns in American political history. It is entirely possible that he may choose to govern using the same strategy of *di vide and conquer*."⁵

III.

In the 1964 speech that launched Ronald Reagan into the political spotlight, he said: "They say the world has become too complex for simple answers. They are wrong."⁶

IV.

In the winter of 1945, Pablo Picasso began work on four series of prints in a Paris lithograph workshop. He worked intensely for several months, producing two series of female heads, a series of nudes, and a series of eleven bull prints. Typically, lithograph prints are made by adding layers of ink to a lithograph stone, pressing a print sheet onto the ink like a stamp, and then

⁴ FiveThirtyEight, "Who will win the presidency?", 2016.

⁵ The Conversation, "5 Things that Explain Donald...", 2016. Emphasis added.

⁶ Reagan, "A Time for Choosing...", 1964.

adding additional layers of ink onto the stone for the next print⁷. In Picasso's time, a lithograph stone could be cleared of ink only a few times between prints before becoming too smudged to produce quality images. Picasso baffled the printmakers who assisted him by re-using a single lithograph stone for each print in a series, marking a stone and then scraping it clean. Even more remarkably, Picasso reversed the traditional lithograph workflow in his bull series by starting with a complete animal and finishing with just a few lines outlining what was left of the bull.

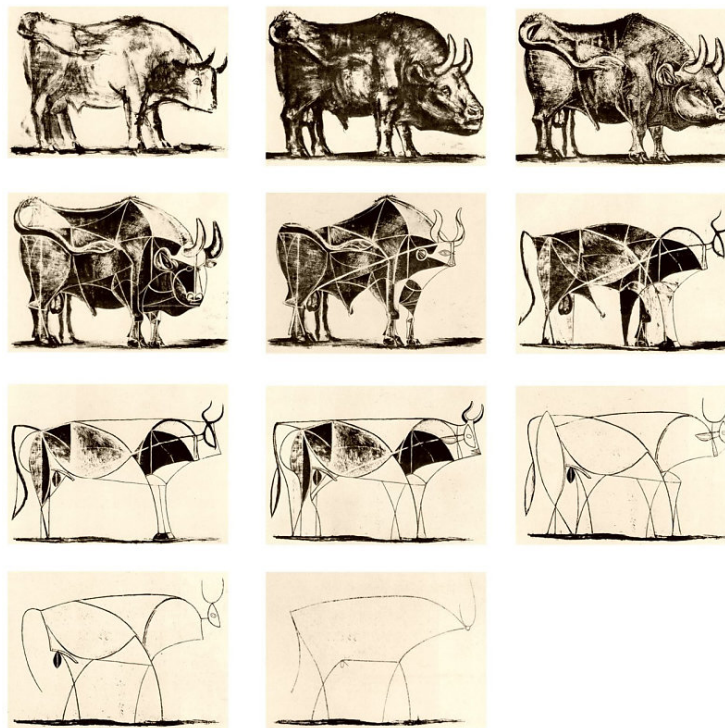


Figure 1: Pablo Picasso, 1945-46. "The Bull." Images are given in the order they were produced, from top to bottom, left to right

⁷ Lithograph printing on stone involves first drawing an image using a greasy ink or crayon on a stone surface. This is then "chemically fixed" and then doused with water. The uncovered regions of the stone absorb the water and then reject the following layer of oil-based ink, which only adheres to the drawn portion of the stone. A print sheet is then pressed onto the stone, flatly printing the oil-ink image onto the sheet (see the Masterworks Fine Art source for a contextual description, from which I have summarized).

Picasso was uniquely interested in the process and progression of his work, meticulously timestamping and signing many of his pieces⁸. His approach broke from the usual artistic paradigm: “A picture used to be a sum of additions. In my [Picasso’s] case, a picture is a sum of destructions. I do a picture—then I destroy it. In the end, though, nothing is lost.”⁹ If you look at the lithograph series print in Figure 1, it’s not difficult to see how Picasso moves from one print to another and iteratively destroys pieces of the bull. While creating these particular prints, he was recorded saying: “Look...we ought to give this bit to the butcher...or this one...” Picasso was “reducing, always reducing” in the process of “seeking his own bull.”¹⁰

If you had shown me the final bull print one year ago, I probably would have smugly pointed out that my nine-year-old brother could have drawn that. Clearly, demonstrating his artistic prowess was not Picasso’s purpose with this print, for he certainly had great skill (and his artwork at the age of nine far surpasses my little brother’s).¹¹ So why did Picasso so aggressively parse down what looks like a finished product in the first few prints? Art Historian Irvin Lavin suggests that Picasso wanted “to retrieve the bull’s constituent parts, to recover and

⁸ Earlier in 1945, speaking of one of his paintings Picasso said, “If it were possible I would leave it as it is, while I began over and carried it to a more advanced state on another canvas. Then I would do the same thing with that one. There would never be a finished canvas, but just the different ‘states’ of a single painting, which normally disappear in the course of the work.” (Lavin, “Picasso’s Bull(s): Art History...”, 78.)

⁹ Ibid., 79.

¹⁰ The full quote: “One day... he started work on the famous bull. It was a superb, well-rounded bull. I thought myself that that was that. But not at all. A second state and a third, still well-rounded, followed. And so it went on. But the bull was no longer the same. It began to get smaller and to lose weight...Picasso was taking away rather than adding to his composition...He was carving away slices of his bull at the same time. And after each change we pulled a proof. He could see that we were puzzled. He made a joke, he went on working, and then he produced another bull. And each time less and less of the bull remained. He used to look at me and laugh. ‘Look...,’ he would say, ‘we ought to give this bit to the butcher. The housewife could say look I want that piece, or this one...’ In the end, the bull’s head was like that of an ant...At the last proof there remained only a few lines. I had watched him at work, reducing, always reducing. I still remembered the first bull and I said to myself: What I don’t understand is that he has ended up where really he should have started! But he, Picasso, was seeking his own bull. And to achieve his one line bull he had gone in successive stages through all the other bulls. And when you look at that line you cannot imagine how much work it involved...” Ibid., 80.

¹¹ Google search Picasso Hercules 1890 and you’ll see what I mean.

reduce the *disjecta membra* of his dream bull—bred of pure lines—to an elemental, disembodied, quintessential bulliness.”¹² For Picasso, the bull was a reoccurring motif laden with symbolic weight, representing St. Luke, patron Saint of artists, the Spanish people, bestiality, virility, and even himself. Picasso seemed to be focused on eliminating this contextual noise and reducing the bull down to its essence, its “pure spirit,” “ideal state,” or *disegno interno*¹³. At its most essential level, Picasso has given us a bull, and in spite of its simplicity, we can all recognize it.

Picasso wasn’t alone in his reductive paradigm; Scandinavian contemporary Piet Mondrian of the *De Stijl* art movement wanted to bring art to the common man by stripping away its pretext and complexity. His characteristic white fields with black grids and blocks of color were intended to be a refuge from the inaccessibility and rigor of the institutional art movement.¹⁴ Mondrian once said: “The appearance of natural form changes, but reality remains. To create pure reality plastically, it is necessary to reduce natural forms to constant elements of form, and natural color to primary color. The aim is not to create other particular forms and colors, with all their limitations, but to work toward abolishing them in the interest of a larger unity.”¹⁵ For these artists, reducing an object to a fundamental essence represented an ultimate form of communication, a universal language that clearly expressed something in its truest sense.

So my father’s simplification of my dating situation was to get to the clearest form of communication, to access a universal language I could understand, right? When we wish to communicate clearly, we reduce to the minimum, as evidenced by the seasoned undergraduate

¹² Lavin, “Picasso’s Bull(s): Art History...”, 80.

¹³ Latin for “inner design,” an artistic ideal. *Ibid.*, 79, 89

¹⁴ For instance, *Composition in Red, Blue and Yellow*

¹⁵ Honor and Fleming, *The Visual Arts: A History*, 806.

student's list of bullet-point classroom notes, a streamlined computer calculator interface, Picasso's bull, and my father's "bottom line." That certainly makes things easier to understand, and frankly is easier to digest than algorithm complexity equations enumerating divide-and-conquer multiplication. It also gives everyone access to the otherwise inaccessible, as Mondrian pointed out. I wonder, though, how safe this mission for simplicity is: are we actually finding a fundamental essence of something, or just ignoring details?

V.

Not long after Karatsuba's breakthrough in algorithmic multiplication, Yasusuke Murakami published in *Econometrica* a formalization of representative governments, a "Formal Structure of Majority Decision." Murakami described, in technical terms, the necessary conditions for a representative system of government to work, building upon seminal work by K. May that had done the same for democracy. Calling the model a "majority decision," Murakami describes a class of "group decision functions" that at least match the complexity of Gauss' equations. Not necessarily attempting to prove something about representative governments, Murakami set out to formalize how democracy works in terms of logic.

For a theoretical system of representative voting to work, Murakami claims that the system must at least be self-dual, monotonic, nondictatorial, and permit each voter only one vote. In English, this means that if every voter in a group reversed their decision, the group's collective decision would be reversed (self-dual); if every voter were to vote again on some candidate either the same or differently, the outcome of the group vote would reflect that

(monotonicity¹⁶); and that no one individual exists in a group of voters that determines the preference of the entire group.

On face value, these assumptions seem obvious. They are also entirely inviable in modern voting. Some voters *do* cast their votes based on what others choose, and the United States Electoral College can cause majority preferences in voting constituencies to be misrepresented at a higher vote.¹⁷ Most of us would agree that the 2/3 supermajority requirement for Congress to override a presidential veto would qualify as “democratic,” but it violates majority decision’s self-duality. Murakami himself cedes that the “concept as well as the reality of democracy is so complex that [this]...attempt is by no means satisfactory.” If that is true, what good does Murakami’s logical voting framework *do*? Conditions that are logically essential for the system *to work don’t work*, so conclusions drawn by the system are not infallible. At its very practical best, democracy seems to be flawed. If I cannot find a democratic system of government that guarantees nondictatorship and autonomy, should I turn my back on the system as a flawed effort?

Silly question—of course not. Even if we use Murakami’s four requirements as a normative standard, there is no way they could all be implemented. And frankly, this isn’t Murakami’s point at all. He concludes with the following comment: “Insofar as the majority principle is regarded as the essence of democracy, our necessary condition gives the minimum

¹⁶ This is an oversimplification: what the paper actually says is that every person’s vote D_i can be represented as either 1, 0, or -1, where 1 is in favor of some candidate over another, 0 is indifferent towards either, and -1 is in favor of the opposing candidate. If each voter’s choice D is matched by some D'_i where D'_i represents a voting decision that is either the same, or ‘less than’ the original D , then the decision of a group of voters (D_1, D_2, \dots, D_m) is either the same or ‘less than’ the group of $(D'_1, D'_2, \dots, D'_m)$.

¹⁷ For instance, this happened in the most recent 2016 election: Hillary Clinton had nearly 3 million more popular votes than Donald Trump, but Trump won the presidency with 306 of the total 538 Electoral College votes. (CNN Politics, “Presidential Results,” 2017.)

requirement for democracy while May's condition presents the maximum. Together, they will specify, the author hopes, the logical expanse of democratic systems."¹⁸ In other words, by simplifying democratic voting systems to four constraints, we learn something about the fundamental character of these systems. In this work, like that of Picasso and Mondrian, simplification distills the character of an idea or principle into something comprehensible.

A few years ago, I spent a summer in eastern Germany doing bioinformatics research at the Max Planck Institute for Chemical Ecology. Those few months were my initiation-by-fire into computational biology research: I was tasked with replicating a genomewide gene regulation and co-expression study for the Institute's model plant, the wild coyote tobacco, a job that could well have been work for three interns. At the conclusion of the internship, my father visited me and took me south to the Bavarian Alps, where we spent a long morning hiking. On the way up, we stopped for lunch: elderberry cream buttermilk and wursts, overlooking miles of lush green rolling mountains. When we finally reached the top of the trail, the view was breathtaking, and I wished I could distill the serene moment and save it, figuratively drink it in and embed it in my mind to counteract the stressful months behind me. I walked a few hundred feet away from my dad to a small rocky drop-off and hid just beneath it, out of sight from other tourists, to enjoy a moment of quiet. I remember trying to breathe as deeply as I could, hoping that maybe that would cement the peaceful moment into my own gene regulation. Just me, the mountains, the trees, bees humming around. Beautiful and peaceful and rejuvenating. Inevitably, each plant I

¹⁸Yasasuke, "Formal Structure of Majority Decision," 717.

sat upon had its own complex gene regulation network just waiting to be analyzed, but that wasn't on my mind anymore.

Ironically, now, a year and half later, the only thing that seems to count to most folks is the genomewide analysis project I did; the graduate school applications I am assembling enumerate the co-expression and motif analysis in as gory detail as I can afford. For my career and bioinformatics skills, the internship was an invaluable ordeal. But I still cringe a little when I remember the head-banging-against-the-wall feeling I had most days. The feelings when I reminisce on the mountaintop vista, however, are entirely opposite, a slight tugging on my heart strings and silent sigh of contentment.

VI.

In the field of computer science, computer scientists use what is called “BigO” notation to compare the efficiency of different algorithms. This notation communicates—in the form of a mathematical function—the running time of a given algorithm in terms of n , the size of a problem to solve, and is written as $\Theta(n)$ or $O(n)$.¹⁹ Typically speaking, algorithms are designed to use as few computer steps as possible and so be as fast as possible.²⁰ For instance, by reducing the number of computer steps, Karatsuba’s divide-and-conquer algorithm improved the BigO complexity of multiplication from $\Theta(n^2)$ to $\Theta(n^{1.58})$ —a small but significant effect.

¹⁹ BigO comes in three flavors that describe the speed of an algorithm: an upper bound (O), a lower bound (Ω) and a close match to the true speed (Θ , used if $O = \Omega$). The “ O ” version is used when referring to an upper bounding function, or the worst an algorithm could be, while the Θ version, which I use here, gives an estimate of what the algorithm’s performance usually is.

²⁰ I say “typically” because on one occasion a high school friend of mine attempted to bypass our school’s computer security system by overclocking his computer’s CPU (running *too many* operations at once). This both damaged the equipment and failed to bypass security.

What makes a new student of computer science algorithms sometimes scratch his or her head at this concept of BigO is that few—if any—algorithms contain only one step that can be captured in a single $\Theta(n)$ or $\Theta(n^2)$ statement. In fact, the principle of BigO is one of significant simplification. If you were to account for each computer function call made in a canonical divide-and-conquer algorithm, the true complexity calculation would be, well, more complex. For multiplication, you would first need to account for all the times you split up the factors to be multiplied into smaller multiplication problems, which you then split up again into smaller problems, and again and again until you reach a “trivial”²¹ multiplication problem that you can easily solve.²² Then you need to accommodate for the cost of recombining answers at each step. In our divide-and-conquer multiplication example, multiplying two numbers gets split up into three easier multiplication problems, each of which are then split up into 3 easier multiplication problems, all the way down until we are just multiplying 1s. This means we are performing $3 + 9 + 27 + \dots$ steps to recombine the multiplied answers, the last step of which is close to $n^{1.58}$ number of recombinations (where n is the size of the numbers we are multiplying). So a more accurate reflection of our run-time would be something like $\Theta(n + 3n + 9n + \dots + n^{1.58})$. But this does not fit BigO syntax, which dictates we remove the constants and terms of lower exponent and report only the largest step of the function. So the algorithm’s complexity is hidden away and we are left with a mere $\Theta(n^{1.58})$.

I share this admittedly long-winded explanation to highlight something that appears contradictory but really isn’t. As we just said, BigO removes constants and smaller steps,

²¹ *Trivial* here means easily solvable, not unimportant. It is usually used this way in computer science, which can lead to confusion when taken out of context.

focusing only on the overarching slowest step. On the other hand: “...algorithm developers are very interested in constants and would gladly stay up nights in order to make an algorithm run faster by a factor of 2.”²³ Yet any discussion of BigO usually concludes with one very simple value, which is reported in paper abstracts, discussed in the classroom, and used for comparison.²⁴ When we *build* algorithms, we want to try and save on every $3n$ and $9n$ operation that we can; but when we *talk* about and *compare* algorithms, we only report the biggest step, $\Theta(n^{1.58})$. Why two disparate mentalities? Having studied BigO throughout my college career, I honestly find myself ignoring this discrepancy. Maybe all the steps of algorithmic runtime are important, but when comparing algorithms, BigO gives an easy comparison, and the best insight into runtime limitations when n becomes massive. From my student’s perspective, all of the details in determining BigO—the $3ns$ and $9ns$ —are only important inasmuch as they help me provide the correct test answer of BigO. So please, hide the details. It’s good enough.

In 2007, my family moved to Hanoi, Vietnam, as part of my father’s assignment in the U.S. Foreign Commercial service. We learned to love many aspects of the Vietnamese culture and people. However, for a long time my mother and I struggled with what we saw as a pervasive indifference to our Western standard of customer service. We would call the store up the street and request a bottled drinking water delivery, only be informed that the driver was taking a nap and wouldn’t be available for a few more hours. In the middle of the workday! And then, if we tried to bargain for some product at the local market, service was hit and miss; if the

²³ Dasgupta, *Algorithms*, 8.

²⁴ Take a look at the paper “A Linear-Time Burrows-Wheeler Transform Using Induced Sorting” by Okanohara and Sadakane if you are feeling bold: the abstract describes a an algorithm boasting linear time ($O(n)$), but then describes multiple $O(n)$ steps, which means a constant is being removed.

vendor didn't like your bargaining terms, they ignored your proposal and turned their noses up. How rude! I thought to myself: this just reveals their laziness and feeling of entitlement, a relic of Ho Chi Minh's communism. They don't know what it means to win customers by quality service.

Some years later, maturity and new knowledge cast my experience in Vietnam in a clearer light. What I had not known was that rice wine was a staple at nearly every Vietnamese meal for the working class. Having never once gone to a Vietnamese Bia Hoi bar and not had access to real street food as a white foreigner, I didn't recognize the prevalence of this beverage nor its effects on the human mental constitution. Of course the Vietnamese were napping in the hot, humid afternoon! Not only drowsy from a lunchtime drink, most had no air conditioning and could not escape the sodden midday heat. This combination would be enough to leave even the most energetic employees in a state of total lethargy. My simplistic and biased former assumptions had left me blind to the obvious tendencies of human nature.

VII.

Murakami's observations about failed voting systems don't sit well with me—I mean, democracy actually works, doesn't it? When I was in high school, I ran for election to student government. Peers in my class voted, and at the end of voting, I had the most votes. Obviously, I was the best candidate, and my peers thought so too; a straightforward plurality vote of beautifully executed democracy, free from any dilemma Murakami suggested. Or so I've told myself for years. Perhaps I have a future in politics; I was, after all, chair of the Senate at the United Nations International School in Hanoi. But if I'm honest, I must admit that plurality rule and a good group of high school friends don't even start to explain what voting is really like on

any other scale. Understanding voting and voter choice seems to be hopelessly complex; I could probably cede this point with just the title of the 2002 Statistical Science article, “Why does voting get so complicated?”

For starters, to just attempt to understand voting systems, we must assume that voters will make “rational” decisions that relate to their preferences. Social scientists have identified five requirements of a rational choice: utility, purposefulness, certainty, sincerity and comparability. As an example, if I were voting on a movie to watch with a group of friends, I would consider the personal satisfaction I receive from watching a given movie (utility) and would prefer a movie that increases that utility (purposefulness). I would probably pick a movie that I know I am likely to enjoy (i.e. a sci-fi over a chick-flick; certainty), and would not pick one film simply to oust another movie option (sincerity). If I prefer a sci-fi over a chick-flick over a thriller, I would prefer the sci-fi over the thriller if those were the only options (comparability).

Given these rational imperatives make sense, you and I have probably violated one or two at some point in our voting lives (I have certainly agreed to watch movies I am less fond of for reasons outside of the utility of the movie). Even political scientists recognize that these assumptions cannot capture the full nuance of human decision-making. In the aforementioned paper, authors Gill and Gaiours begin their discussion of voter systems by saying: “we generally rely on these assumptions in the descriptions of voting systems and voting behavior.... [b]ut deliberately avoid the controversy about broadly assigning universal individual rationality...”²⁵ So for the sake of argument, let’s ignore the known complexity of human decision making and say it’s as simple as voter rationality.

²⁵ Gill and Gaiours, "Why Does Voting Get So Complicated?", 385.

Before proceeding, I want to point out that this quotation highlights an important point about making points: for Gill and Gainous to present their analysis of voting complexity and democratic participation, they are forced to simplify the reality of voter decisions into something digestible. By way of analogy, they are ignoring each of the small $3n$ and $9n$ operations of human existence in favor of the overall basic decision-making component of $\Theta(n^{1.58})$. Like both Picasso and Murakami, they shave away details to reduce to the fundamentals and make a point.

Gill and Gainous then present a smorgasbord of vote aggregation systems, describing no fewer than ten ways of grouping people's votes, formally called "preference aggregation."²⁶ Each of these systems has different strengths and weaknesses; some favor the overall electorate, others the absolute majority, and others the candidates that are the overall least objectionable. For instance, approval voting grants voters up to one vote for each candidate, allowing voters to vote for multiple candidates and enhancing the strategic power of each voter's preferences. But encouraging strategic voting could violate the fundamental assumption of voter sincerity, introducing a break in the presupposed logic.

In fact, Kenneth Arrow won a Nobel Prize for drawing a related conclusion more broadly: all implemented vote aggregations systems (called social welfare functions) have some logical defect. In order for a voting system to successfully aggregate the preferences of voters, it must fulfill conditions that are "logically impossible" to all meet at the same time.²⁷ This

²⁶ Others they describe include cumulative voting, plurality rule, approval voting, Condorcet voting, Coombs procedure voting and anti-plurality voting, just to name a few. I certainly don't claim to have a working understanding of most of these.

²⁷ The four conditions are: unrestricted domain, in which voters must be able to have any preference over outcomes; independence of irrelevant alternatives, which requires that preferences between alternative candidates is only based on their relative ordering; the pareto principle, which states that if all voters prefer a certain candidate that one is the preferred candidate; and non-dictatorship. (Gill and Gainous, "Why Does Voting Get So Complicated?", 393.)

theorem has therefore been called the “impossibility theorem”: there is no means of grouping votes that would “yield a truly democratic system” in the logical sense

So is democracy broken, Mr. Arrow? What about my high school election? If this analysis reveals anything, it is that voting systems are far from simple. My naïve and sanguinely simplistic view has been disabused of its innocence by a proper understanding of complexity. I can’t help but chuckle at all of this: so if you want to make a point about something and see what you want to see—a fast algorithm, a logical voting system, palatable art—you should simplify everything and distill it. Conversely, if you want to see something different that disrupts your perceptions, put in all the details and consider every angle. Don’t be fooled by the eight arching lines of Picasso’s bull: there’s a lot more baggage than you think. And just because the President says he hears your vote doesn’t mean he actually does; it might just be the result of a flawed social welfare function. How often do you and I think about that? Sometimes it’s easier not to

IIX.

Fareed Zakaria, a journalist for CNN, hosted a CNN Special Report entitled “Why Trump Won.” In his report, he condensed Trump’s victory down to the “4 C’s” that divided America and gave Trump the upset victory: capitalism, culture, class, and communication. According to Zakaria, the Republican Party had better exploited these differences than the Democrats, placing them in their current position of power in both the executive and legislative branches of government. Ending on a somber tone, Zakaria said: “...the real victory will come for this country when someone looks at these deep forces that are dividing it and tries to construct a politics that will bridge them. Rather than accept that America must remain a country split between two tribes...he or she would speak in a language that unites them. That kind of

leadership would win not just elections—but a place of honor in American history.”²⁸ That’s pretty simple, right? Just unite and conquer, don’t divide and conquer. Maybe that’s even better than Karatsuba’s $\Theta(n^{1.58})$ algorithm.

I was recently reading a blog post relating the principle of Occam’s razor to computer algorithm design. ²⁹ In his own words, Mr. Occam said, “Plurality should not be posited without necessity”—basically, don’t give multiple options, answers, explanations, or solutions or outcomes if you don’t need to. For algorithm architect and blogger Michael Lant, this means make as few assumptions as possible because they are “unknowns masquerading as knowns.” In biological language, Lant compares this to the principle of parsimony, which says that things usually behave in the simplest and most economical way possible.³⁰ I read this and smirked. Doesn’t this contradict the ever-present tug of entropy, the mysterious ΔS that affects every chemical reaction, the formation of stars and what my bedroom looks like by Friday afternoon? Why is it that humans and everything else tend to chaos, but we can only comprehend and feel at ease with our world when we’ve reduced it to a single BigO, or some socio-economic explanation, or a logically faultless voting framework?

IX.

²⁸ Fareed, *Why Trump Won*, 2017.

²⁹ Michael Lant’s blog on “Software Development, Agile Methods and the Intersection of People Process and Technology.”

³⁰ The first hit on a Google Search gave this definition: ‘the scientific principle that things are usually connected or behave in the simplest or most economical way, especially with reference to alternative evolutionary pathways.’ Search performed October 30, 2017.



Figure 2: Duane Hansen's "Man on a Mower (Edition 1/3)".

What can happen when we take off our figurative mental blinders and put all the details back into something? Consider hyper and photo-realism, art movements that emerged in the late 1960s and were committed to portraying things in extraordinary detail to create the illusion of reality. Duane Hanson's fiberglass sculptures are a well-known example: look at Figure 2, a photograph of Hanson's *Man on a Mower (Edition 1/3)*. I was almost forced to do a double- take when I saw this—this could easily be my neighbor in Georgia on his John Deere in the

middle of a summer day, only instead he is sitting in a museum somewhere. Duane's most well-known works were almost exclusively of human subjects, and earlier in his career they addressed topics of social injustice and violence. Later on, his subjects became more commonplace and mundane as he depicted typical middle and lower-class Americans. "Why *not* look at this guy sitting right next to me, what's going on, what I see on the TV and in the newspaper...?"³¹ Hanson once posited. In fact, his artwork was so unambiguous and pointed that a piece commenting on abortion received the following critical evaluation: "...we do not consider [this] a work of art... We find the subject objectionable, and continue to wish that such works which merely attempt to express experience in the raw could be referred to by some other name."³²

³¹ Buchsteiner, "Art is Life and Life is Realistic," 75. Emphasis added.

³² *Ibid.*, 73

Hanson's sculptures capture trivial³³ subject matter with "pedantic" and "almost impertinent" attention to detail.³⁴ But why bother to artistically recreate the minutiae of average things when are so seemingly unimportant? After a few moments interacting with the expressionless lawnmowing man, the purpose becomes clearer; Hanson's work forces us to confront mundane daily details that we otherwise might ignore. Then the question: why *do* we ignore those details? Perhaps our brains assume they are irrelevant—in the massive stream of sound and sight and smell, the man on the lawnmower just doesn't take priority. Or perhaps we are indifferent to the details. Or perhaps we know thinking about them will show us something we might not like to see. Said Hansen, "My work deals with people who lead lives of quiet desperation. I show the empty-headedness, the fatigue, the aging, the frustration. These people can't keep up with the competition. They're left out, psychologically handicapped."³⁵

Interacting with these figures, viewers look for a reaction, some kind of recognition from the statue, but get none, only a tired stare. In a sense, this isn't so different from what happens every day in our impersonal society. We all just pass by the man on his lawnmower, the women at the grocery store, the businessman on the street corner, without giving a second thought. When confronted with these individuals in vivid detail and "the resignation, emptiness, and loneliness of their existence [that] captures the true reality of life for these people..." we can no longer ignore what we see.³⁶

Flipping through books with photographs of Hanson's works, I can't help but think that most of these folks look like losers. Usually looking forlorn, bored, unhealthy or overweight,

³³ Does the computer science or traditional literary definition fit best here, I wonder?

³⁴ Buchsteiner, "Art is Life and Life is Realistic," 77.

³⁵ Ibid, 69.

³⁶ Ibid, 77.

they are perpetually trapped in absolutely trivial tasks. But I guess that is Hanson's whole point, isn't it? Look at these people who you think are losers. Here they are in exquisite detail—the lines on their faces, the brand of clothes they wear, right down to their human emotions. Have you ever noticed? What challenges me most about Hanson's work is that I can sense some of my own human experience in the fiberglass forms; the feeling of being trapped in a redundant cycle of the same, of feeling defeated by the system. I might step past and ignore these people, dismissing them as losers, but in so doing simultaneously condemn myself. Hansen said it himself: "I'm not duplicating life, I'm making a statement about human values."³⁷ Facing me with the full depth and painful detail of these people, I am forced to reevaluate my own critique of them.

X.

Hansen's art reveals that an examination of detail may teach us something unexpected about ourselves and our worldview. A closer look can shatter our simplified perceptions and reveal incongruences in our assumptions. This paradigm-shifting experience is certainly not limited to the field of art; in fact, it is even mirrored in computer science by a class of algorithms that seem to overturn the foundations of BigO. Consider linear programming, a family of algorithms that attempts to "optimize"³⁸ an outcome based on a set of linear constraints. What makes this group so bizarre is that within it some algorithms have "better" (lower) BigO complexity but run more *slowly* than those with *worse* (larger) BigO.³⁹

³⁷ Buchsteiner, "Art is Life and Life is Realistic," 69.

³⁸ Either minimizing or maximizing some result, like revenue or production levels.

³⁹ This is only one layer of the "paradox of linear programming," which initially began as a conundrum being able to solve something *practically* but not *theoretically*. (Dasgupta, *Algorithms*, 220.)

To understand how this is possible, consider the classic “linear problem” of a chocolate shoppe that wants to maximize its revenue. The store sells three different types of chocolate—a high-end, middle-end, and low-end variety—which they sell at prices of \$12, \$6, and \$2, respectively. The store naturally wants to maximize profit and usually cannot sell more than 100 bars of chocolate on any given day. Furthermore, because of equipment limitations, only 35 of the high-end bars can be produced on a given day, 75 of the middle-end, and the total number of high and low-end chocolate bars cannot exceed 40. Linear programming would *optimize* chocolate production to maximize profits by identifying how many of each bar should be made on a given day. While this example is pretty simple, imagine a larger company with thousands of products and thousands of constraints. As the problem size grows, this becomes much less *trivial* to solve intuitively and requires a strong algorithmic solution.

Since 1947, the canonical approach to solving this problem is the simplex algorithm, which basically graphs each linear condition⁴⁰ on a coordinate plane and finds all the intersection points of the graphed conditions that match the constraints. The computer systematically marches through these points until it gets to the point representing optimal chocolate-production levels for maximum revenue.⁴¹ This gives a roughly exponential BigO runtime (something like $O(2^n)$), which is *really* not good. ⁴² Usually, programs with this kind of BigO are considered effectively unsolvable on large scales.

⁴⁰ A linear condition might be, for instance, setting the number of high-end-chocolates ≤ 35 , which excludes all values over 35.

⁴¹ Simply put, the algorithm calculates the revenue value of an intersection point and its neighboring intersection points and then moves to the neighbor with a higher revenue value. This traversal is repeated until the highest value is reached

⁴² This description is a simplification: technically speaking, the operation of finding neighboring points is $O(mn)$ for a problem of m variables and n linear constraints, which is repeated at most $O\left(\frac{(n+m)!}{n!m!}\right)$ times. Mathematically speaking, we usually group algorithms with factorial (!) runtime in with exponential runtime—very slow.

Then in 1979, Soviet mathematician Leonid Khachiyan formalized the “ellipsoid algorithm,” which solved the same chocolate-shoppe problem while boasting polynomial BigO complexity, a substantial improvement over simplex’s exponential BigO.⁴³ While Khachiyan’s work was a significant theoretical breakthrough for linear programming, actual runtime comparisons of the two algorithms revealed that the simplex approach was markedly and unexpectedly faster. Mathematically, Khachiyan’s algorithm should blow the simplex approach out of the water, so explains this unexpected outcome? The oddly consistent practical speed of the simplex algorithm over the years has led to its own “folklore” that posits linear time performance.⁴⁴ This folklore was even validated by randomized tests that revealed monotonic⁴⁵ functions influencing linear performance for the simplex algorithm. At the end of the day, ⁴⁶ it seems like the simplified BigO runtime calculation used to evaluate these algorithms brushes away too many details, leaving a misleading impression of how slow (or fast) simplex actually is.⁴⁷

I think I’ve identified a common thread in these spaces where simplicity fails and complexity reigns. For both Murakami’s government and for simplex performance, *monotonicity* is one of the common words that appears. While I doubt the burden of failure can be laid on this graduate-level vocabulary term, the foreignness of this word highlights that the search for simplicity is constantly being undermined by an ever-present arch nemesis, complexity. In this

⁴³ Khachiyan’s algorithm forms ellipsoid shapes around the possible optimization points and, using a series of complex separation steps, reduces the size of the ellipsoid repeatedly until it narrows down onto the point that gives the best revenue. The key step in this algorithm runs in $O(mn)$ time, repeated a polynomial number of times until the optimal answer is found. (Arora, “The Ellipsoid Algorithm for Linear Programming,” 2005.)

⁴⁴ Shamir, “The Efficiency of the Simplex Method...”, 306.

⁴⁵ Here’s that word again—this time it’s referring to a function that is either always increasing or always decreasing. Interestingly, its not too dissimilar to the earlier definition of monotonicity in footnote 15 above.

⁴⁶ Like my father, here I am trying to lay claim to the bottom line.

⁴⁷ To prove my conclusion: a 2004 paper introducing a new algorithm analysis approach called “smoothed analysis” explains that “the simplex algorithm has smoothed complexity polynomial in the input size and the standard deviation of Gaussian perturbations.” Sufficiently cryptic, I’d say. (Spielman and Teng, “Smoothed Analysis of Algorithms...”, 2004.)

noble quest to condense life, science, government, art, and existence to something comprehensible, some adversary is indefatigably resistant. (Like we say, the devil is in the details.) Sometimes we can learn something from the details; sometimes we are stumped and left with nothing better than nebulous folklore. It's as simple as comparing BigOs or casting your vote when you know what you are hoping to see. But the moment you take into account the true realities of human failure and computer runtime, these simplifications aren't so useful anymore. Our perception of the arbitrary is disrupted as we take a closer look at the true performance of linear programming algorithms, at the motionless man on the lawnmower stuck in his $O(\infty)$ lawnmowing algorithm, get confused at an unexpected presidential race outcome, or at the chronic motion of our lives. It seems to me that while simplicity might reveal the essential existence or character of something, the details reveal that that essence isn't everything we thought it was.

XI.

Many have connected Picasso's bull renditions with cave murals discovered around the same time, and Picasso himself marveled at the beautiful simplicity of this ancient art. Picasso's secretary Jaime Sabartes recalled Picasso's awe with primitive sculpture when he said:

How do you explain to yourself...the disappearance of this marvelous simplicity?

This is due to the fact that man ceased to be simple. He wanted to see farther and so he lost the faculty of understanding that which he had within reach of his vision...The same happens with a watch...the moment it falls into the hands of a watchmaker... His manipulations will rob it of its purity, and this will never

return. It may preserve the same eternal appearance, just as the idea of art subsists, but... Its essence has evaporated...⁴⁸

In fact, art historians suggest Picasso spent years trying to *unlearn* his years of artistic training, going from the detail of his juvenile *Hercules*⁴⁹ to the naked bull lithograph. In effect, he was moving from complexity in his art to simplicity.

XII.

Is that it then, Picasso? The world, art, and everything else is just simple, like a beautiful watch, and it's not until human hands attempt to make it their own does it become defiled and convoluted? What does that mean for me in my human experience: how do I make sense of my non-simple existence? I am a highly complex biological being that is regulated by constant biochemical feedback systems and gene co-expression networks; a sentient, thinking being whose behavior is shaped by the psychological experience of daily living; a sexual being with hormonal chemistry; a being of habits and molecules that tend towards entropy and disorder. I am surrounded by cultural norms that guide and misguide my assumptions and expectations about people; I live in a high-speed information-saturated world that tells me to do it all; I look for scapegoats to my problems, but insist on receiving all the praise for my hard work. I crave a breath of fresh air to clear my mind, only to return to the chaos. I am full of complexity, and if I let myself think about it, I just can't escape it.

Simplicity certainly exists, somewhere out there, or maybe, at least, in my and Picasso's heads. In that simplicity, I think we both hope to find the true bull, the pure democracy, the perfect algorithm, the essence of our complex reality. Every day, I either consciously or

⁴⁸ Lavin, "Picasso's Bull(s)...", 83-84.

⁴⁹ See reference 11 above.

subconsciously ignore the details of life around me. I push the details away, hoping to somehow make sound, good choices without being overwhelmed by every crease on the man in the lawnmower's face and every $9n$ and $3n$ consideration. We say that "the devil is in the details," so if we ignore the details, it's harder to see him and we retain our sanity. But in those devilish details we find our simplifications undermined, nuanced, deepened, and abolished. In fact, it is often in those details that we learn the most and find the biggest breakthroughs, like Gauss and Karatusba showed us in computer science. At times, we wish we could do without those details and persist in living simply, a simple democracy devoid of Arrow's paradox. But when our simple models fail, it becomes clear that we need both simplicity and complexity: one to survive with our sanity, and the other to elucidate our hidden inadequacies.

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